Exam IV, Fall 2006

Name:_____

Signature:_____

Show all your work and give reasons for your answers. Good luck!

Part I All problems in part I are worth 15 points each.

(1) Find the angle between the vectors $< 1, 2, 3 >$ and $< -1, 0, 1 >$. Also find the area of the parallelogram spanned by these vectors.

(2) Find the equation of the plane which goes through the point $(1, 3, -2)$ and is is perpendicular to the line

$$\begin{cases} x = 1 + t \\ y = -1 - t \\ z = 2 - 3t \end{cases}$$

(3) Find the volume of the slanted box spanned by the vectors $< 1, 1, 1 >$, $< -1, 0, 1 >$ and $< 0, 2, 1 >$. Do these vectors lie in a common plane?

(4) Find the intersection of the line

$$\begin{cases} x = 1 - t \\ y = -1 + 2t \\ z = 2 - 3t \end{cases}$$

and the plane $2x + y - z = 5$.

(5) Find the line of intersection of the planes $x - y + z = 1$ and $-2x + y + 3z = 2$.

Part II The problem in Part II is worth 25 points.

(6) Let

$$\ell_1 = \begin{cases} x = 1 + 2t \\ y = -1 + 4t \\ z = 2 - 3t \end{cases}$$

and

$$\ell_2 = \begin{cases} x = 3s \\ y = 4 - 2s \\ z = 2 + 5s \end{cases}$$

be two lines.

(a) If the lines ℓ_1 and ℓ_2 intersect, find the point of intersection. Otherwise find the distance between them.

(b) Are the lines ℓ_1 and ℓ_2 skew?

(Use the back of the page to work this problem.)