# MA 125-6C, CALCULUS I

November 29, 2006

Name (Print last name first): .....

Student ID Number: ..... ..... .....

## TEST IV

## PART I

**Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!**

Each question is worth 5 points.

Question 1

Find all the critical numbers of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$.

Answer: .....

Question 2

The function $f(x) = x^2 - 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$. Find the number $c$ that satisfies the conclusion of the Mean Value Theorem.

Answer: .....

Question 3

Find the absolute maximum value of the function $f(t) = t^2 + t$ on the closed interval $[-3, 1]$.

Answer: .....

Question 4

Find the open interval on which the function $g(x) = x^5 - 5x + 2$ is decreasing.

Answer: .....

Question 5

Find the open interval on which the function $h(x) = xe^x$ is concave up.

Answer: .....

Question 6

Find the most general antiderivative of the function $f(x) = 1 + e^x + \cos x$.

Answer: .....

**PART II**

Each problem is worth 14 points.

**Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.**

## Problem 1

Suppose that the derivative of a function $f(x)$ is given by

$$f'(x) = (x-2)^2(x+2)^3(x-3)^4.$$

Answer all the following questions.

(a) Find all the critical numbers of the function $f(x)$.

(b) On what interval(s) is the function $f(x)$ increasing? (Justify your answer!)

(c) On what interval(s) is the function $f(x)$ decreasing? (Justify your answer!)

## Problem 2

Consider the function

$$f(x) = x - \frac{1}{3}x^3.$$

Answer all the following questions.

(a) Find the (open) interval of increase, and all the (open) intervals of decrease.

(b) Find all local maximum and minimum values.

(c) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up.

(d) Find the inflection point. [Be sure to give the $x$ and the $y$ coordinate!]

(e) Use the information from parts (a)–(d) to sketch the graph.

## Problem 3

(1) Find the dimensions of a rectangle with perimeter 100 ft and whose area is as large as possible.

(2) Find two positive numbers whose product is 64 and whose sum is minimum.

## Problem 4

(a) Find the most general antiderivative of the function

$$f(x) = 12x^2 - 8x^{3/5} + 3\sqrt{x}.$$

(Simplify your answer!)

(b) Find the most general antiderivative of the function

$$f(x) = \frac{1}{x} + e^x + \frac{1}{1+x^2}.$$

## Problem 5

An object moves along a straight line with acceleration

$$a(t) = \sin t + \cos t.$$

Use antiderivatives to answer the following questions.

(a) Find the velocity function $v(t)$ of the object if its initial velocity is $v(0) = 2$.

(b) Find the position function $s(t)$ of the object if its initial position is $s(0) = 0$.