

Calculus I Test 3. Nov. 2005 NAME:.....

**Justify your answers. You may not use calculators, books, or notes. Use the methods of this course to justify all conclusions You must CIRCLE answers (except for graphs). Failure to follow these instructions on any problem will result in a zero score for that problem.**

1.) The radius of a circle is increasing at the rate of  $3\text{cm}/\text{sec}$ . At what rate is the area increasing when the radius is  $5\text{cm}$ ?

2.) Let  $f(x) = x^4 - 8x^2$ . Use the methods of calculus to find the  $(x, y)$  coordinates of:

a.) All local minima of  $f$ .

b.) All local maxima of  $f$ .

3.) Let  $g(x) = 4x + \frac{3}{x}$ . Find the absolute maximum and minimum values of  $g$  on  $[\frac{1}{2}, 2]$ .

4.) Find  $\lim_{x \rightarrow 0} \frac{1-e^x}{\sin x}$

5.) Find the general antiderivative of  $F(x) = \frac{1}{x^2} - \frac{1}{x}$ .

6.) Find two numbers  $x$  and  $y$  whose difference is 6 and whose product is a minimum

7.) A street light is mounted at the top of a 15ft. tall pole. A boy 5 ft. tall walks away from the pole at a speed of  $3ft/sec$  along a straight path. How fast is the tip of his shadow moving when he is 40 ft. from the pole?

9.) Find  $\lim_{x \rightarrow 0} \frac{\cos(7x) - \cos(3x)}{x^2}$

8) Let  $g(x) = x^3 + 6x^2 + 9x$ .

a) Find all open intervals on which  $g$  is increasing.

b.) Find all local minima and maxima of  $g$ .

c) Find all open intervals on which  $g$  is concave up

d.) Sketch the graph of  $g$  showing clearly all important features (such as extrema, concavity, inflection points).

10.) A box with a square base and an open top is to have a volume of 72 ft.<sup>3</sup>. Find the dimensions of the box that minimize the amount of material used.

11. A particle moves along a straight line with acceleration  $a(t) = 12t + 4$  (in  $m/s^2$ ). Its initial velocity is  $v(0) = 3 m/s$  and its initial displacement is  $s(0) = 5 m$ . Find its position after  $t$  seconds.

12.) Find the point on the line  $y = 5x + 2$  that is closest to the origin.

Extra Credit: Find the largest right circular cylinder that can be inscribed in a sphere of radius  $R$ .