MA 125 Test 1 Sept. 2005. NAME_____

You may not use calculators, notes, or books. Do your own work.

Justify and simplify your answers mathematically. 'Show your work.' **CIRCLE ANSWERS.**

A. In each of the following, find the limit, if it exists. Infinite limits are allowed. If a limit fails to exist, so state [5 each].

1. $\lim_{x\to\infty} \frac{7-3x+5x^2}{9x^2+4x+2}$

2. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

3. $\lim_{x\to 2} \frac{x+2}{x-2}$

4. $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$

5. $\lim_{x\to 5^+} \frac{1-x}{x-5}$

6. $\lim_{x \to 3^-} \frac{x^2-9}{|x-3|}$

7. $\lim_{x \to 5} \frac{x^2-7x+10}{x^2-6x+5}$

8. $\lim_{x \to \infty} (\sqrt{4x^2+x} - 2x)$

B. Follow instructions for each of the following (10 points each).

9. Let $g(x) = \frac{1}{\sqrt{x}}$. Use the definition of derivative to find $g'(a)$ for $a > 0$.

10. Sketch the graph of a function F that satisfies: $\lim_{x\to-\infty} F(x) = 3$, $\lim_{x\to 2^-} F(x) = \infty$, $\lim_{x\to 2^+} F(x) = -\infty$, $\lim_{x\to 5^-} F(x) = 1$, $\lim_{x\to 5^+} F(x) = 4$, $\lim_{x\to\infty} F(x) = -2.5$, $F(5) = 3$.Also, F is defined and continuous at all points except possibly not at $x = 2$ and $x = 5$.

11. Sketch the graph of a continuous function g that satisfies:
(i) $g(x) = 0$ for $x = -7, -3, 2, 6$.
(ii) $g'(x) > 0$ on $(-\infty, -5)$ and on $(0, 4)$; $g'(x) < 0$ on $(-5, 0)$ and on $(4, \infty)$.
(III) $g''(x) < 0$ on $(-\infty, -3)$ and on $(2, \infty)$; $g''(x) > 0$ on $(-3, 2)$
and (iv) $g'(x) = 0$ for $x = -5, 0, 4$.

12. The displacement (in meters) of an object moving in a straight line is given by $s = 3t^2 + 2t + 1$, where t is measured in seconds.

(A) Find the average velocity over the time periods

(i) $[1, 4]$

(ii) $[1, 2]$

(B) Find the instantaneous velocity at $t = 1$.

13. Let $g(x) = x^4 + 8x^{-1} + 1$

(A) Find the derivative $g'(x)$

(B) Find the x coordinate of every point on the graph of g at which the tangent line is horizontal.

(C) Find the equation of the line tangent to the graph of g at the point $(1, 10)$.

14. (A.) The graph below depicts the graph of a function f. On the same set of axes sketch a graph of its derivative function f′.

(B.) The graph below depicts the graph of the derivative g′ of a function g. On the same set of axes sketch a graph of the function g, assuming $g(-2.5) = 0$.

Extra Credit. (10 points) A rock is tossed straight up from the surface on the planet S-57. While in its trajectory the height (in meters) of the rock is given by $s = 16t - 2t^2$, where t is in seconds. Find the velocity at time t and answer the questions.

(A) When does the rock reach its maximum height?

(B.) How high does the rock go?