Math 126 FINAL EXAM

Do not use any books or notes. You can use a calculator, but not graphing calculator. If you use a calculator, leave your results in exact form instead of decimal form. **Show all work for full credit.**

1. Evaluate the definite integral: (20 points)

(a) $\displaystyle\int_1^2 \frac{x^2+1}{\sqrt{x}}\,dx$ (b) $\displaystyle\int_0^2 (x^2+1)^2\,dx$ (c) $\displaystyle\int_0^{\pi/2} e^{\sin x}\cos x\,dx$

(d) $\displaystyle\int_1^2 x\sqrt{x-1}\,dx$ (e) $\displaystyle\int_0^{1/2} \sin^{-1} x\,dx$

2. Evaluate the indefinite integral: (20 points)

(a) $\int \sqrt[5]{3-5x}\,dx$ (b) $\int \frac{e^x}{e^x+1}\,dx$ (c) $\int \sqrt{x}\ln x\,dx$

(d) $\int x\tan^{-1}x\,dx$ (e) $\int \cos x \ln(\sin x)\,dx$

3. Determine whether the SEQUENCE converges or diverges. (9 points)

(a) $a_n = \dfrac{2^n}{3^{n+1}}$ (b) $a_n = \dfrac{2n-3}{3n+4}$ (c) $a_n = \dfrac{\ln n}{\sqrt{n}}$

4. Determine whether the SERIES is convergent or divergent. If it is convergent, find the SUM. (6 points)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$

5. Determine whether the SERIES is convergent or divergent. You do NOT need to find the sum. But you need to support your claim by appropriate work. (20 points)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^2+1}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-1}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3-n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

6. Find the RADIUS of convergence and INTERVAL of convergence of the **power series** $\sum_{n=1}^{\infty}(-1)^n \frac{(x+2)^n}{n2^n}$. (5 points)

7. Find $\mathbf{a}-\mathbf{b}$ and $3\mathbf{a}+4\mathbf{b}$: (4 points)

(a) $\mathbf{a}=\mathbf{i}-2\mathbf{j}+\mathbf{k},\ \mathbf{b}=\mathbf{j}+2\mathbf{k}$

(b) $\mathbf{a}=3\mathbf{i}-2\mathbf{k},\ \mathbf{b}=\mathbf{i}-\mathbf{j}+\mathbf{k}$

8. Find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$. (6 points)

$$\mathbf{a} = \langle -3, 2, 2 \rangle, \qquad \mathbf{b} = \langle 6, 3, 1 \rangle$$

9. Sketch the region enclosed by the curves $y = 1 + \sqrt{x}$ and $y = 1 + x/3$. Decide whether to integrate with respect to x or y. Then find the area of the region. (4 points)

10. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about $x = 2$. (6 points)