Math 126 TEST III

Do not use any books or notes. You can use a calculator, but not graphing calculator. If you use a calculator, leave your results in exact form instead of decimal form. **Show all work for full credit.**

1. Determine whether the SEQUENCE converges or diverges. If it is convergent, find the LIMIT. (15 points)

   (a) $a_n = \dfrac{n+1}{3n-1}$ (b) $a_n = \dfrac{n}{1+\sqrt{n}}$ (c) $a_n = \dfrac{\ln(n^3)}{n}$

2. Determine whether the SERIES is convergent or divergent. If it is convergent, find the SUM. (18 points)

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^{n-1}}+\frac{2}{3^{n-1}}\right)$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5+2^{-n}}$ (c) $\displaystyle\sum_{n=1}^{\infty}\left[\sin\left(\frac{1}{n}\right)-\sin\left(\frac{1}{n+1}\right)\right]$

3. Determine whether the SERIES is convergent or divergent. You do NOT need to find the sum. But you need to support your claim by appropriate work. (16 points)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$

4. For each series determine whether it is convergent or divergent, and whether it is ABSOLUTELY convergent. Support your claim by appropriate work. (16 points)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

5. Find the RADIUS of convergence and INTERVAL of convergence of the **power series**. (20 points)

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (b) $\sum_{n=0}^{\infty} \frac{n^2 x^n}{10^n}$

6. Find a POWER SERIES representation for the function $f(x) = \dfrac{x}{1-2x}$ and then determine the RADIUS of convergence. (8 points)

7. Find the Maclaurin series for $f(x) = \cos x$ using the definition $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$. (7 points)