MA 126, Fall 2004

TEST # 3

December 2, 2004 (105 minutes)

Name: SSN:

Max. Points: 100 + 8 Bonus Points: Test Grade:

This test is closed book and closed notes. You may use a calculator. Make sure to show all your work, including necessary explanations on how you arrive at your results. Write on back or use extra sheets if needed.

1. Find the radius of convergence and the interval of convergence for each of the following two power series (8P+6P):

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$

(b) $\sum_{n=1}^{\infty} \frac{(x-2)}{2^n n!}$

2. Find the Taylor polynomial of second order $T_2(x)$ for $f(x) = \frac{1}{x}$ at $a = 3$. (8P)

3. Find a power series for $x \tan^{-1} x$ and use it to calculate $\int_0^{1/2} x \tan^{-1} x \, dx$ up to an error of less than 0.001. (8P)

4. Find the Mclaurin series for $\dfrac{x^2 - \sin x^2}{x^3}$. (6P)

5.* Use your knowledge on Taylor series to find the sum of the series (8P*)

$$\sum_{n=2}^{\infty} \frac{x^{2n+1}}{n!}.$$

6. Show that the equation $x^2 + y^2 + z^2 = x - 2z - 1$ represents a sphere in space. What are its center and radius? (5P)

7. For the vectors $\mathbf{a} = \langle 2, 1, -1 \rangle$ and $\mathbf{b} = \langle -3, 2, 1 \rangle$ find (3P+4P+4P)

(a) $\mathbf{a} \cdot \mathbf{b}$

(b) The angle between $\mathbf{a}$ and $\mathbf{b}$ in degrees.

(c) The vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

8. A force of 20N acts in the direction S40°E, i.e. in the direction which is 40° east from the southern direction, and moves an object 200m to the east. How much work is done? (5P)

9. If $\mathbf{a} \times \mathbf{b} = 2\mathbf{j}$, what is (3P+3P+3P)

(a) $\mathbf{a} \times 2\mathbf{a}$

(b) $(\mathbf{b} \times \mathbf{a}) \times \mathbf{i}$

(c) the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$?

10. Find parametric equations for the line that passes through the points $(-1, 2, 5)$ and $(2, -1, 3)$. (5P)

11. (a) Find a unit vector which is orthogonal to the plane through the points $P(1, 2, 3)$, $Q(2, 3, 4)$ and $R(1, 3, 5)$. (6P)

(b) Find an equation for the plane which passes through the points P, Q and R from part (a). (3P)

(c) Do the points P, Q, R from Part (a) and the point $S(3, 2, 1)$ lie in the same plane? (4P)

12*. Find parametric equations for the line of intersection of the two planes $x + y + z = 2$ and $x - y - 2z = 1$. (8P)

13. Provide the formulas which express the cartesian coordinates x, y and z of a point in space in terms of its spherical coordinates ρ, θ and ϕ. Also illustrate these relations geometrically. (8P)