TEST # 2

October 26, 2004 (105 minutes)

Name: SSN:

Max. Points: 100 + 12 Bonus   Points: Test Grade:

In all problems, for getting credit do not only state the result, but also the methods used to get this result (like tests for convergence/divergence, etc.). Make sure to show all your work. Write on back or use extra sheets if needed.
No book, no notes, and no calculator are to be used!

1. Calculate the area enclosed between the two curves $y = x^2$ and $y = 4 - x^2$. (10P)

2. The region which is enclosed by $y = x$, $y = 2$ and $x = 4$ is revolved about the x-axis. Find the volume of the resulting solid. (12P)

3. Find the length of the graph of the function $y = \frac{2}{3}x^{3/2}$, where $0 \le x \le 3$. (6P)

4. Determine the length of the parametric curve (6P)

$$x = 2t + 3, \qquad y = 3t + 2, \qquad 1 \le t \le 3$$

5. Do the following limits exist? If yes, find their value. (4P+4P+4P+4P)

(a) $\lim\limits_{n\to\infty} \dfrac{n^2 + 3n - 2}{5n^2}$

(b) $\lim\limits_{n\to\infty} 2\left(\dfrac{-3}{4}\right)^n$

(c) $\lim\limits_{n\to\infty} \dfrac{1 + (-1)^n}{n}$

(d) $\lim\limits_{n\to\infty} \dfrac{(\ln n)^2}{n}$

6. Find the sum of the following series. (6P+6P+6P*)

(a) $\sum\limits_{n=1}^{\infty} \dfrac{2}{3^{n-1}}$

(b) $\sum\limits_{n=1}^{\infty} \dfrac{1 - 3^n}{6^n}$

(c)* (Bonus) $\sum\limits_{n=2}^{\infty} \dfrac{1}{n^2 - 1}$

7. Do the following series converge or diverge? Give reasons for your answers.
(6P+6P+6P+6P+6P+6P*)

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3 - 2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{1 + n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\cos n)}{n^2}$

(f)* (Bonus) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^{n^2} + 1}$

8. Find the smallest n such that the first n terms of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

approximate the sum of the series up to an error of less than 0.01. (8P)