Calculus I

FINAL EXAM

Version A

December 13^{th}, 2004

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. *(12 pts)* Evaluate the following limits:

(a) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

(b) $\lim_{x\to \pi} \frac{3}{\sin^2 x}$

(c) $\lim_{x\to 0^+} (1+3x)^{\frac{1}{x}}$ {Hint: use the fact that $f(x) = e^{\ln f(x)}$}

2. *(10 pts)* Find the equation of the tangent line to the graph of $y = f(x) = \frac{x}{x+2}$ at the point (-1,-1).

3. *(8pts)* Differentiate

$$x^2 \ln x + e^{x^2}$$

4. *(8pts)* Differentiate

$$\cos(x + 2x^2)$$

5. *(10pts)*Find y' using implicit differentiation if $2xy + x^2 = e^y$.

6. *(14pts)* Use calculus to determine intervals of increase or decrease, local maxima/minima, intervals of concavity and inflection points of the function $f(x) = \frac{1}{5}x^5 + \frac{1}{2}x^4 - x^3 + 2$. (You may round off the y-coordinate of the inflection point to 2 decimals). Use this information to sketch the graph of $f(x)$.

7. (a) *(7pts)* Let $f(x) = \sqrt[3]{4x-4}$. Find the linear approximation of $f(x)$ at $x = 3$.

(b) *(3pts)* Use the above linear approximation to estimate $f(2.97)$.

8. *(10pts)* Find the point on the line $y = 2x + 2$ that is closest to the point $(5, 2)$.

9. *(8pts)* Find the most general antiderivative of:

$$\frac{2}{\sqrt{1-x^2}} + \cos x + e^x$$

10. *(10pts)* Evaluate the following integral:

$$\int_1^3 \frac{1}{3x} + 3x^2 \, dx$$

(BONUS, 2pts) Calculate the derivative of $f(x) = x^2$.

(BONUS, 3pts) Calculate $\int_{-2}^{2} f(x)\,dx$ if $f(x)$ is given by

$$f(x) = \begin{cases} \sqrt{4-x^2} & \text{if } -2 \leq x \leq 0 \\ 2x+2 & \text{if } x \geq 0 \end{cases}$$