Calculus I

TEST 3A

December 6^{th}, 2004

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. *(10pts)* At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing east at 25 km/h. How fast is the distance between the ships changing at 2:00 P.M.?

2. *(10pts)* Find the absolute maximum and absolute minimum values of $f(x) = \ln(x^2 - 6x + 11)$ on the interval $[0, 4]$.

3. *(20pts)* Find intervals of increase or decrease, horizontal and vertical asymptotes, local maxima/minima, intervals of concavity and inflection points of the function $f(x) = \frac{x}{x^2-1}$. Use this information to sketch the graph of $f(x)$.

4. Evaluate the following limits:

a) *(5pts)*

$$\lim_{x \to 0} \frac{x^3}{\sin x - x}$$

b) *(5pts)*

$$\lim_{x \to \infty} x^2 e^{-x}$$

5. *(10pts)* Find the equation of the line through the point $(2, 3)$ that cuts off the least area from the first quadrant.

6. (a) *(4pts)* Find the most general antiderivative of the function $f(x) = 3e^x + \cos(2x) + 3\sin(x)$. Check your answer by differentiating the answer you obtain.

(b) *(6pts)* Find $g(x)$ when $g''(x) = x^2 + 3$, $g(1) = 2$ and $g'(2) = 1$.

7. *(10pts)* Using right endpoints, write a Riemann sum for

$$\int_{-1}^{5} \ln(x^2 + 1)\, dx$$

dividing the interval $[-1, 5]$ up in six subintervals. You do <u>not</u> have to evaluate this sum.

8. Evaluate the following integrals:

a) *(6pts)*

$$\int_0^1 \frac{2}{1+x^2} + x^2 \, dx$$

b) *(6pts)*

$$\int_{-2}^{5} |2t - 3| \, dt$$

9. *(8pts)* Find the area enclosed by the parabola $y = x^2 - 5x + 6$ and the x-axis. (Hint: sketch the parabola).

10. *(BONUS, 5pts).* Find the area enclosed by the parabola $f(x) = x^2 - 2x + 2$ and the line $g(x) = 2x - 1$.