MA 227 , Calculus - III. THE FINAL EXAM Monday, December 8, 2003.

Student's Name ____

. *(Please, print)*

GIVE REASONS FOR YOUR ANSWERS!

CODE:

I. (10%) Find the length of the curve:

$$\vec{r}(t) = (2\sin t, 5t, 2\cos t), \quad -10 \leq t \leq 10.$$

II. (10%) Find the equation of the tangent plane to the surface $z = y^2 - x^2$ at the point on the surface corresponding to $x = 0, y = 2$.

III. (10%) Find the linearization of the function $f(x, y) = e^x \cos(xy)$ at the point $(0, 0)$. Use the linearization to calculate $f(0.01, -0.02)$.

IV. (10%) Use the chain rule to find $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial s}$:

$$u(x, y) = xy + yz + zx,$$

$$x = st, \quad y = e^{st}, \quad z = t^2.$$

V. (10%)
a) Find the gradient of $f(x, y, z) = xy^2z^3$ at the point $P(1, -2, 1)$.
b) Find the derivative of the function in the direction of the vector $\vec{u} = \frac{1}{\sqrt{3}}(1, -1, 1)$ at the same point P.

VI. (10%) Find the maximum value of the directional derivative of the function

$$f(x, y, z) = x + \frac{y}{z},$$

at the point $(4, 3, -1)$ and the direction in which it occurs.

VII. (10%) Find local maxima (if any), local minima (if any) and saddle points (if any) of the function:

$$f(x, y) = 4xy - x^4 - y^4.$$

VIII. (10%) Find

$$\int_D \int ye^x dA,$$

where D is a triangle region with vertices $(0,0)$, $(2,4)$, $(6,0)$.

IX. (10%) Find the volume of the solid bounded with the cylinder $x^2 + y^2 = 1$, the plane $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$.

X (10%). Use spherical coordinates to find the volume of a sphere of radius a.

XI. (extra credit 12%) Find

a) (6%) $\int_C \sqrt{x^2+y^2}ds$,

b) (2%) $\int_C \sqrt{x^2+y^2}dx$,

c) (2%) $\int_C \sqrt{x^2+y^2}dy$,

d) (2%) $\int_C \sqrt{x^2+y^2}dz$,
the curve C being described by the formulas:

$$x(t) = 4\cos t, \quad y(t) = 4\sin t, \quad z = 3t,$$

$$-2\pi \leq t \leq 2\pi.$$