Mathematics 126
Midterm 2
Oct. 17, 2003

• Calculators are allowed *only* for numerical calculations.

• There are two sheets of scratch paper attached at the end of the exam. Use them and but do not tear them off the exam in doing so, and hand them in together with the exam.

• Show your work; clearly write down each step in your calculations/reasonings to be considered for full/partial credit.

1. A log 10-ft long is cut at 1-foot interval and its cross sectional areas $A(x)$ (at a distance x-ft from the end of the log) are listed as below. Use the midpoint rule with $N = 5$ to estimate the volume of the log.

x(ft)	$A(x)$(ft^2)	x(ft)	$A(x)$(ft^2)
0	0.68	6	0.53
1	0.65	7	0.55
2	0.64	8	0.52
3	0.61	9	0.50
4	0.58	10	0.48
5	0.59		

2. Evaluate the following improper integrals. The value of the integrals may be $+\infty$ or $-\infty$.

a)

$$\int_1^{\infty} \frac{1}{1+x^2}\, dx.$$

b)

$$\int_2^{6} \frac{1}{\sqrt{x-2}}\, dx.$$

3. Determine whether the following integral is convergent or divergent using the Comparison Theorem.

$$\int_1^\infty \frac{1}{\sqrt{x^4+1}}\,dx.$$

4. Find the area bounded by $y^2 = x$, $x - 2y = 3$.
Hint: first draw the graphs and decide which axis (x or y) would be better to integrate along.

5. Consider a box with a square base of side length 2, of height 1. Inside the box a solid hemisphere of radius 1 is fixated to the bottom of the box. Let $V(h)$ be the volume of water in the box whose depth is h. $(0 \leq h \leq 1)$

a) Calculate the area $A(x)$ of the cross section when you slice the container at the height x. $(0 \leq x \leq 1)$

b) Write down the function $V(h)$ as an integral *and* evaluate when $h = 1$.